cosine integral, $\mathrm{Ci}\,(x)$

$$\mathrm{Ci}\,(x) = -\int_x^{\infty} \frac{\cos(y)}{y}\,dy = \gamma + \ln(x) + \sum_{n=1}^{\infty} \frac{(-x^2)^n}{2\cdot n\cdot(2\cdot n)!}$$

where γ is Euler-Macheroni constant ($\approx 0.577215664901532861$).

difference of cosine integrals

for $\alpha > 0$, $\beta > 0$ and $z \to +0$

$$\mathrm{Ci}\,(\alpha\cdot z) - \mathrm{Ci}\,(\beta\cdot z) = \ln(\alpha\cdot z) - \ln(\beta\cdot z) + \mathrm{O}\,(z^2) \approx \ln\left(\frac{\alpha}{\beta}\right)$$

integral expressed by the difference of cosine integrals

for $a > 0$, $b > 0$, $c < d$ and $c \to +0$

$$\int_c^d \frac{\cos(a\cdot x) - \cos(b\cdot x)}{x}\,dx$$

$$= \int_{a\cdot c}^{a\cdot d} \frac{\cos y}{y}\,dy - \int_{b\cdot c}^{b\cdot d} \frac{\cos y}{y}\,dy$$

$$= \int_{a\cdot c}^{\infty} \frac{\cos y}{y}\,dy + \int_{\infty}^{a\cdot d} \frac{\cos y}{y}\,dy - \int_{b\cdot c}^{\infty} \frac{\cos y}{y}\,dy - \int_{\infty}^{b\cdot d} \frac{\cos y}{y}\,dy$$

$$= -\mathrm{Ci}\,(a\cdot c) + \mathrm{Ci}\,(a\cdot d) + \mathrm{Ci}\,(b\cdot c) - \mathrm{Ci}\,(b\cdot d)$$

$$\approx \mathrm{Ci}\,(a\cdot d) - \mathrm{Ci}\,(b\cdot d) - \ln\left(\frac{a}{b}\right)$$

band pass Shepp filter, $h_S(r)$**, for differential phase contrast**

for $0 \le z_1 < z_2 \le z_N$ (z_N : Nyquist frequency) and $\Pi = \dfrac{\pi}{2\cdot z_N}$

$$h_S(r) = \frac{2\cdot z_N}{\pi^2}\cdot\int_{z_1}^{z_2} \frac{1}{z}\cdot\sin\left(\frac{\pi}{2}\cdot\frac{z}{z_N}\right)\cdot\sin(2\cdot\pi\cdot r\cdot z)\,dz$$

$$= -\frac{1}{2\cdot\pi}\cdot\frac{1}{\Pi}\cdot\left[\mathrm{Ci}\,(|2\cdot\pi\cdot r+\Pi|\cdot z) - \mathrm{Ci}\,(|2\cdot\pi\cdot r-\Pi|\cdot z)\right]_{z_1}^{z_2}$$

$$= -\frac{1}{2\cdot\pi}\cdot\frac{1}{\Pi}\cdot\left\{\begin{array}{l} \mathrm{Ci}\,(|2\cdot\pi\cdot r+\Pi|\cdot z_2) - \mathrm{Ci}\,(|2\cdot\pi\cdot r-\Pi|\cdot z_2) \\ -\mathrm{Ci}\,(|2\cdot\pi\cdot r+\Pi|\cdot z_1) + \mathrm{Ci}\,(|2\cdot\pi\cdot r-\Pi|\cdot z_1) \end{array}\right\}$$

when $z_1 \to +0$

$$h_S(r) \approx -\frac{1}{2\cdot\pi}\cdot\frac{1}{\Pi}\cdot\left\{\mathrm{Ci}\,(|2\cdot\pi\cdot r+\Pi|\cdot z_2) - \mathrm{Ci}\,(|2\cdot\pi\cdot r-\Pi|\cdot z_2) - \ln\left|\frac{2\cdot\pi\cdot r+\Pi}{2\cdot\pi\cdot r-\Pi}\right|\right\}$$

when $z_2 = z_N$ and $r = \dfrac{j}{2\cdot z_N}$ (j : integer)

$$h_S\left(\frac{j}{2\cdot z_N}\right) \approx \frac{z_N}{\pi^2}\cdot\left\{\mathrm{Ci}\left(\frac{\pi}{2}\cdot|2\cdot j-1|\right) - \mathrm{Ci}\left(\frac{\pi}{2}\cdot|2\cdot j+1|\right) + \ln\left|\frac{2\cdot j+1}{2\cdot j-1}\right|\right\}$$