値 1 の線分の像に Gaussian filter を施した像の値の空間分布、$h(r)$

- r　座標値もしくは座標原点からの距離
- R　線分の長さの半値 (half length of line segment)
- L　1 次元 Gaussian filter のパラメータ (Gaussian radius)

$$f(x) = \begin{cases} 0 & \text{when } |x| > R \\ 1 & \text{when } |x| \le R \end{cases}$$

$$g(l) = \frac{1}{\sqrt{\pi}\cdot L}\cdot \exp\left(-\frac{l^2}{L^2}\right)$$

$$\begin{aligned} h(r) &= \int_{-\infty}^{+\infty} f(x)\cdot g(r-x)\cdot dx \\ &= \frac{1}{\sqrt{\pi}\cdot L}\cdot \int_{-R}^{+R} \exp\left(-\frac{(r-x)^2}{L^2}\right)\cdot dx \\ &= \frac{1}{\sqrt{\pi}\cdot L}\cdot L\cdot \int_{(-R-r)/L}^{(+R-r)/L} \exp(-t^2)\cdot dt \\ &= \frac{1}{2}\cdot\left(\operatorname{erf}\left(\frac{+R-r}{L}\right) - \operatorname{erf}\left(\frac{-R-r}{L}\right)\right) \\ &= \frac{1}{2}\cdot(\operatorname{erf}(m) + \operatorname{erf}(p)) \quad \text{where} \begin{cases} m = (R-r)/L \\ p = (R+r)/L \end{cases} \end{aligned}$$

$$\operatorname{erf}(s) \equiv \frac{2}{\sqrt{\pi}}\cdot\int_0^s \exp(-t^2)\cdot dt$$

	$g(l) =$	$h(r) =$
line segment	$\frac{1}{(\sqrt{\pi}\cdot L)^1}\cdot\exp\left(-\frac{l^2}{L^2}\right)$	$\frac{1}{2}\cdot(\operatorname{erf}(m) + \operatorname{erf}(p))$
circle	$\frac{1}{(\sqrt{\pi}\cdot L)^2}\cdot\exp\left(-\frac{l^2}{L^2}\right)$	$1 - Q\left(\sqrt{2}\cdot\frac{r}{L}, \sqrt{2}\cdot\frac{R}{L}\right)$
sphere	$\frac{1}{(\sqrt{\pi}\cdot L)^3}\cdot\exp\left(-\frac{l^2}{L^2}\right)$	$\frac{1}{2}\cdot\left(\operatorname{erf}(m) + \operatorname{erf}(p) - \frac{1}{\sqrt{\pi}}\cdot\frac{L}{r}\cdot\left(\exp(-m^2) - \exp(-p^2)\right)\right)$

$$Q(a, b) \equiv \int_b^{\infty} t\cdot\exp\left(-\frac{1}{2}\cdot(t^2 + a^2)\right)\cdot I_0(a\cdot t)\cdot dt$$: Marcum Q-function

$I_0(s)$: 0-th order modified Bessel function of the first kind