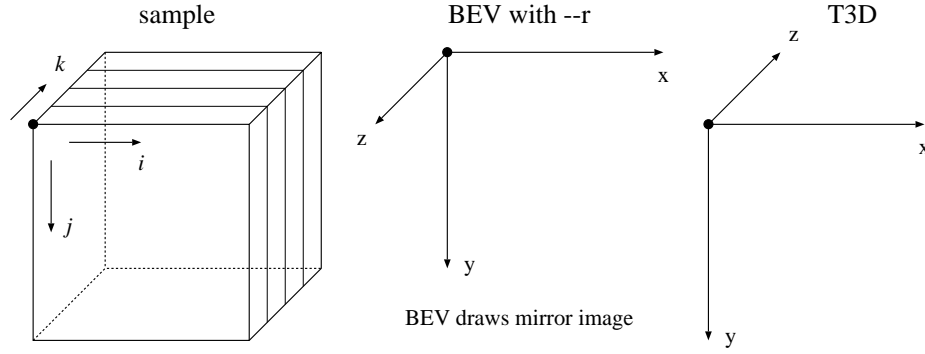


coordinate systems



transform formula used in BEV with --r (exactly speaking, $\theta = 0$ for BEV)

$$\begin{pmatrix} h_1 \\ v_1 \\ d_1 \end{pmatrix} = \Theta^{-1} \Phi^{-1} \Lambda^{-1} \begin{pmatrix} i \\ j \\ -k \end{pmatrix} = \Theta^{-1} \Phi^{-1} \Lambda^{-1} \begin{pmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & -1 \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

$$\Lambda^{-1} = \begin{pmatrix} -\sin \lambda, & \cos \lambda, & 0 \\ -\cos \lambda, & -\sin \lambda, & 0 \\ 0, & 0, & 1 \end{pmatrix} : \text{rotation of angle } \lambda + 90^\circ \text{ around the z-axis fixed on the sample}$$

$$\Phi^{-1} = \begin{pmatrix} 1, & 0, & 0 \\ 0, & \sin \phi, & \cos \phi \\ 0, & -\cos \phi, & \sin \phi \end{pmatrix} : \text{rotation of angle } 90^\circ - \phi \text{ around the x-axis fixed on the sample}$$

$$\Theta^{-1} = \begin{pmatrix} -\cos \theta, & -\sin \theta, & 0 \\ \sin \theta, & -\cos \theta, & 0 \\ 0, & 0, & 1 \end{pmatrix} : \text{rotation of angle } \theta - 180^\circ \text{ around the z-axis fixed on the sample}$$

$$\Theta^{-1} \Phi^{-1} \Lambda^{-1} = \begin{pmatrix} \cos \lambda \sin \phi \sin \theta + \sin \lambda \cos \theta, & \sin \lambda \sin \phi \sin \theta - \cos \lambda \cos \theta, & -\cos \phi \sin \theta \\ \cos \lambda \sin \phi \cos \theta - \sin \lambda \sin \theta, & \sin \lambda \sin \phi \cos \theta + \cos \lambda \sin \theta, & -\cos \phi \cos \theta \\ \cos \lambda \cos \phi, & \sin \lambda \cos \phi, & \sin \phi \end{pmatrix}$$

transform formula used in T3D

$$\begin{pmatrix} h_2 \\ v_2 \\ d_2 \end{pmatrix} = Z_m Y_m X_m \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

$$X_m = \begin{pmatrix} 1, & 0, & 0 \\ 0, & \cos X, & -\sin X \\ 0, & \sin X, & \cos X \end{pmatrix} : \text{rotation of angle } X \text{ around the x-axis fixed on the space}$$

$$Y_m = \begin{pmatrix} \cos Y, & 0, & \sin Y \\ 0, & 1, & 0 \\ -\sin Y, & 0, & \cos Y \end{pmatrix} : \text{rotation of angle } Y \text{ around the y-axis fixed on the space}$$

$$Z_m = \begin{pmatrix} \cos Z, & -\sin Z, & 0 \\ \sin Z, & \cos Z, & 0 \\ 0, & 0, & 1 \end{pmatrix} : \text{rotation of angle } Z \text{ around the z-axis fixed on the space}$$

$$Z_m Y_m X_m = \begin{pmatrix} \cos Y \cos Z, & \sin X \sin Y \cos Z - \cos X \sin Z, & \cos X \sin Y \cos Z + \sin X \sin Z \\ \cos Y \sin Z, & \sin X \sin Y \sin Z + \cos X \cos Z, & \cos X \sin Y \sin Z - \sin X \cos Z \\ -\sin Y, & \sin X \cos Y, & \cos X \cos Y \end{pmatrix}$$

BEV draws mirror image

$$\begin{pmatrix} h_2 \\ v_2 \\ d_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ -v_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} 1, & 0, & 0 \\ 0, & -1, & 0 \\ 0, & 0, & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ v_1 \\ d_1 \end{pmatrix} \rightarrow \begin{pmatrix} 1, & 0, & 0 \\ 0, & -1, & 0 \\ 0, & 0, & 1 \end{pmatrix} \Theta^{-1} \Phi^{-1} \Lambda^{-1} \begin{pmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & -1 \end{pmatrix} \equiv Z_m Y_m X_m$$

$$\begin{pmatrix} \cos \lambda \sin \phi \sin \theta + \sin \lambda \cos \theta, & \sin \lambda \sin \phi \sin \theta - \cos \lambda \cos \theta, & \cos \phi \sin \theta \\ -\cos \lambda \sin \phi \cos \theta + \sin \lambda \sin \theta, & -\sin \lambda \sin \phi \cos \theta - \cos \lambda \sin \theta, & -\cos \phi \cos \theta \\ \cos \lambda \cos \phi, & \sin \lambda \cos \phi, & -\sin \phi \end{pmatrix} \equiv$$

$$\begin{pmatrix} \cos Y \cos Z, & \sin X \sin Y \cos Z - \cos X \sin Z, & \cos X \sin Y \cos Z + \sin X \sin Z \\ \cos Y \sin Z, & \sin X \sin Y \sin Z + \cos X \cos Z, & \cos X \sin Y \sin Z - \sin X \cos Z \\ -\sin Y, & \sin X \cos Y, & \cos X \cos Y \end{pmatrix}$$

T3D \rightarrow BEV : $(X, Y, Z) \rightarrow (\lambda, \phi, \theta)$

from (3, 3) elements of the above equation of matrix

$$\sin \phi = -\cos X \cos Y$$

assuming $|\phi| \leq 90^\circ$ ($\cos \phi \geq 0$)

if $\cos X \cos Y = \pm 1$ ($\sin X = 0$ and $\sin Y = 0$)

$$\phi = \mp 90^\circ \text{ (} \cos \phi = 0 \text{)}$$

from (1, 1) and (1, 2) elements

$$\sin(\lambda \mp \theta) = \cos Y \cos Z$$

$$\cos(\lambda \mp \theta) = \cos X \sin Z$$

assuming $\theta \equiv 0$

$$\tan \lambda = \frac{\cos Y \cos Z}{\cos X \sin Z}$$

else ($\cos \phi > 0$)

from (3, 2) and (3, 1) elements

$$\tan \lambda = \frac{\sin \lambda \cos \phi}{\cos \lambda \cos \phi} = \frac{\sin X \cos Y}{-\sin Y}$$

from (1, 3) and (2, 3) elements

$$\tan \theta = \frac{\cos \phi \sin \theta}{\cos \phi \cos \theta} = \frac{\cos X \sin Y \cos Z + \sin X \sin Z}{-\cos X \sin Y \sin Z + \sin X \cos Z}$$

BEV \rightarrow T3D : $(\lambda, \phi, \theta) \rightarrow (X, Y, Z)$

from (3, 1) elements

$$\sin Y = -\cos \lambda \cos \phi$$

assuming $|Y| \leq 90^\circ$ ($\cos Y \geq 0$)

if $\cos \lambda \cos \phi = \pm 1$ ($\sin \lambda = 0$ and $\sin \phi = 0$)

$$Y = \mp 90^\circ \text{ (} \cos Y = 0 \text{)}$$

from (2, 3) and (2, 2) elements

$$\sin(X \pm Z) = \cos \phi \cos \theta$$

$$\cos(X \pm Z) = -\cos \lambda \sin \theta$$

assuming $Z \equiv 0$

$$\tan X = \frac{\cos \phi \cos \theta}{-\cos \lambda \sin \theta}$$

else ($\cos Y > 0$)

from (3, 2) and (3, 3) elements

$$\tan X = \frac{\sin X \cos Y}{\cos X \cos Y} = \frac{\sin \lambda \cos \phi}{-\sin \phi}$$

from (2, 1) and (1, 1) elements

$$\tan Z = \frac{\cos Y \sin Z}{\cos Y \cos Z} = \frac{-\cos \lambda \sin \phi \cos \theta + \sin \lambda \sin \theta}{\cos \lambda \sin \phi \sin \theta + \sin \lambda \cos \theta}$$